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Phonon effects on a multichannel resonance-level model

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Abstract. The phonon effects on a multichannel resonant-level model have been studied by a bosonization technique and canonical transformations. We found that the model can be mapped onto a single-channel Kondo model and the non-Fermi-liquid transition is modified by local electron–phonon interaction. At strong interaction, only one channel of conducting electrons is necessary in order to realize this transition and it is the phonon effects that control the behaviour of the system.

1. Introduction

There has been increasing interest in non-Fermi-liquid (non-FL) behaviour in strongly correlated electron systems; as a notable example, a phenomenological marginal-FL model, proposed by Varma *et al* [1], describes successfully the normal state of the copper oxide based high-T_c superconductors. Much experimental evidence of non-FL behaviour has also been reported for several heavy-fermion systems [2]. Thus at present, one central issue is how to derive universal non-FL behaviour from strong electron correlations. In particular, it is of great interest to study those models whose properties can be obtained either exactly or within controllable approximations, exhibiting a transition from FL to non-FL behaviour as some parameters of the models are adjusted.

Recently the generalized Anderson model [3, 4] and its spinless version, the multichannel resonant-level (MCRL) model [5, 6, 7] have been studied to show the possible FL–non-FL transition. It is well known that the single-channel resonant-level model can be derived from an antiferromagnetic Kondo model through Tomonaga–Luttinger bosonization and it exhibits the local FL behaviour below the Kondo temperature [8]. The generalization of the single-channel resonant-level model through considering finite-range Coulomb repulsion leads to a coupling of the local electron to other orbital channels, as well as the channel which has the same symmetry as the local orbital. The MCRL model Hamiltonian therefore contains new ingredients other than the usual single-channel resonant-level model, which in fact is a multichannel x-ray edge model displaying a local non-FL behaviour [9]. The essential physics of the MCRL model thus reflects the competition between the FL-type resonant-level model and the non-FL-type x-ray edge model. Some authors have demonstrated that the MCRL model can be transformed to a single-channel (SC) Kondo model. With redefinition of parameters the MCRL model can be mapped onto a ferromagnetic or an antiferromagnetic Kondo model, which respectively correspond to FL or non-FL behaviour. There are two ways [6, 7] to realize this mapping. One concerns the similarity of their partition functions [6], while the other considers Hamiltonian mapping at operator level

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[7]. In fact the FL or non-FL behaviour in the MCRL model depends on fluctuation of the local electron. If it tends to overlap with conduction electrons, i.e., if the local electron is screened by conduction electrons, then the FL behaviour emerges; in contrast, if the local electron fluctuation is separated from the electron gas we have a non-FL degenerate ground state. In the MCRL model one can prohibit the fluctuation of the local electron by means of increasing the channel number. It has been proven that in order to observe the non-FL behaviour the channel number must be at least *three* [6], in the case of sufficiently strong repulsive interactions with the local electron. It is not clear, however, whether the multichannel model is necessary, since a one-band Hubbard model has been proposed to describe the physics of the copper oxide based high-T_c superconductors [10].

The above physical picture of FL and non-FL behaviour stems from strong electron–electron correlations; however, it is well known that there is another way to prohibit fluctuation of the local electron, namely by considering electron–phonon interactions, which play a remarkable role in theories of superconductivity [11] and mixed-valence systems [12]. In the following we intend to discuss the FL–non-FL transition when there exist not only multichannel electron–electron correlations but also electron–phonon interactions. The key result we obtained in this paper is that, at the strong interaction limit, only *one* channel of conducting electrons is necessary to get the FL–non-FL transition and *it is the phonon effects, rather than the electron correlation, that controls the behaviour of the system.*

The arrangement of the paper is as follows: in section 2, we raise the model Hamiltonian including phonon effects and then bosonize it to a simple form. In sections 3 and 4, we map the model Hamiltonian to the SC Kondo Hamiltonian by comparing their partition functions and establishing equivalence at operator level. Finally, we give our discussions and conclusions in section 5.

2. The model Hamiltonian and its bosonization

There are two kinds of electron–phonon interaction: conduction electron–phonon interaction and local electron–phonon interaction. The former can be incorporated into the electronic kinetic energy, leading only to mass renormalization in the mean-field approximation, so we ignore it and only study the latter. Moreover, as we show in the appendix, when we consider the local electron coupling to the nearest-neighbouring ions and only retain longitudinal long-wavelength acoustical phonons, we can treat the three-dimensional phonons as one-dimensional phonons. In addition, allowing for a single impurity it is sufficient to consider only the radial motion of the conduction electrons in each channel; this is equivalent to a one-dimensional problem in half-space $x > 0$, or to an another representation which retains the left-moving electrons and allows x to range over the whole space $-\infty < x < \infty$. After the above considerations we write the following model Hamiltonian ($\hbar = 1$):

$$\begin{aligned} H &= H_0 + H_{e-e} + H_{e-p} \\ H_0 &= iv_F \sum_l \int_{-\infty}^{\infty} \mathrm{d}x \Psi_{l,L}^{\dagger}(x) \frac{\mathrm{d}}{\mathrm{d}x} \Psi_{l,L}(x) + \frac{u_p}{2} \int_{-\infty}^{\infty} \left[\Pi^2(x) + (\bigtriangledown\phi)^2\right] \mathrm{d}x \\ H_{e-e} &= j\left[\Psi_{0,L}^{\dagger}(0)d + d^{\dagger}\Psi_{0,L}(0)\right] + \sum_l V_l \Psi_{l,L}^{\dagger}(0)\Psi_{l,L}(0)\left(d^{\dagger}d - \tfrac{1}{2}\right) \\ H_{e-p} &= \lambda \frac{\partial}{\partial x}\phi(0)\left(d^{\dagger}d - \tfrac{1}{2}\right). \end{aligned} \tag{1}$$

In the above, H_0 describes kinetic energy of conduction electrons and phonons, $\Psi_{l,L}^{\dagger}(x)$ and $\Psi_{l,L}(x)$ are spinless left-moving conduction electron fields with channel index l, $\phi(x)$

denotes phonon field and $\Pi(x)$ is the corresponding momentum field; H_{e-e} describes the hybridization and screening between conduction electrons and local electron, d and $d^\dagger$ operators correspond to a localized orbital, as required by symmetry, the localized impurity hybridizes only with one channel, $l = 0$; H_{e-p} describes the local electron–phonon interactions, λ is the coupling coefficient, and u_p denotes phonon velocity. When we retain only the first term in H_0 and H_{e-e}, it is just the usual MCRL model Hamiltonian. Defining a bosonic field $\Phi_{l,L(R)}(x)$

$$\Phi_{l,L(R)}(x) = \frac{1}{2}\Big[\phi_l(x) \pm \int_{-\infty}^{x} \Pi_l(x')\mathrm{d}x'\Big]$$

where field $\phi_l(x)$ and its conjugate momentum $\Pi_l(x)$ satisfy the standard commutation relation, we can represent left- (right-) moving electrons as follows [12]:

$$\Psi_{l,L(R)}(x) = \frac{1}{\sqrt{2\pi a}} \mathrm{e}^{\mp \mathrm{i}\sqrt{4\pi}\Phi_{l,L(R)}(x)}. \tag{2}$$

Then the bosonic form of the Hamiltonian (1) is

$$\begin{aligned} H_b = {} & (4\pi\rho)^{-1} \sum_l \int \big[\Pi_l^2(x) + (\nabla\phi_l)^2\big]\mathrm{d}x \\ & + \frac{j}{\sqrt{2\pi a}}\Big[\mathrm{e}^{\mathrm{i}\sqrt{4\pi}\Phi_{0,L}(0)} d + d^\dagger \mathrm{e}^{-\mathrm{i}\sqrt{4\pi}\Phi_{0,L}(0)}\Big] \\ & + \frac{1}{\sqrt{\pi}} \sum_l V_l \frac{\partial}{\partial x}\Phi_{l,L}(0)\big(d^\dagger d - \tfrac{1}{2}\big) \\ & + \lambda \frac{\partial}{\partial x}\phi(0)\big(d^\dagger d - \tfrac{1}{2}\big) + \frac{u_p}{2}\int \big[\Pi^2(x) + (\nabla\phi)^2\big]\mathrm{d}x \end{aligned} \tag{3}$$

where $\rho = (2\pi v_F)^{-1}$ is the density of states at the Fermi energy, and a^{-1} is the momentum cut-off. For convenience, we have included the right-moving electrons in the first term of the Hamiltonian (3) with the understanding that they have no effect on the dynamics of the original Hamiltonian. Employing the unitary transformation

$$U_1 = \exp\left\{\mathrm{i}\left[\sqrt{4\pi}\rho \sum_l V_l \Phi_{l,L}(0) + \frac{\lambda}{u_p}\int_{-\infty}^{0} \Pi(y)\mathrm{d}y\right]\big(d^\dagger d - \tfrac{1}{2}\big)\right\}$$

and rotating the boson fields $\Phi_{l,L}(0)$ to their principal axes as

$$\begin{pmatrix} \tilde{\Phi}_{0,L} \\ \tilde{\Phi}_{1,L} \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} = \overline{\overline{W}} \begin{pmatrix} \Phi_{0,L} \\ \Phi_{1,L} \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

we find that the boson fields $\tilde{\Phi}_{l,L}(0)$ ($l > 0$) decouple from the local orbital when the first line elements of the unitary matrix W are chosen as

$$\begin{aligned} W_{00} &= (1 - \delta_0/\pi)(1+\gamma)^{-1/2} \\ W_{0l} &= -(\delta_l/\pi)(1+\gamma)^{-1/2} \quad (l > 0) \end{aligned}$$

with

$$\gamma = -2\frac{\delta_0}{\pi} + \sum_l \Big(\frac{\delta_l}{\pi}\Big)^2. \tag{4}$$

The phase shift of the electrons in the lth channel is given by $\delta_l = \pi\rho V_l$. Then, the Hamiltonian (2) becomes

$$\tilde{H}_b = U_1 H_b U_1^\dagger = (4\pi\rho)^{-1}\int\left[\tilde{\Pi}_0^2(x) + (\nabla\tilde{\phi}_0(x))^2\right]\mathrm{d}x + \frac{u_p}{2}\int\left[\Pi^2(x) + (\nabla\phi(x))^2\right]\mathrm{d}x$$
$$+\frac{j}{\sqrt{2\pi a}}\left\{\exp\left(\mathrm{i}\left[\sqrt{4\pi}(1+\gamma)^{1/2}\tilde{\Phi}_{0,L}(0) - \frac{\lambda}{u_p}\int_{-\infty}^{0}\Pi(y)\mathrm{d}y\right]\right)d + \mathrm{h.c.}\right\} \quad (5)$$

in which the free scalar fields $\tilde{\phi}_l$ ($l > 0$) have been omitted. It deserves to be pointed out that if we set $\lambda = 0$ in the above expression, we obtain the transformed solution about the usual MCRL model after the same steps [7], which is expressed as

$$\tilde{H}_{b0} = (4\pi\rho)^{-1}\int\left[\tilde{\Pi}_0^2(x) + (\nabla\tilde{\phi}_0(x))^2\right]\mathrm{d}x$$
$$+\frac{j}{\sqrt{2\pi a}}\left\{\exp\left(\mathrm{i}\left[\sqrt{4\pi}(1+\gamma)^{1/2}\tilde{\Phi}_{0,L}(0)\right]\right)d + \mathrm{h.c.}\right\}. \quad (6)$$

In the following we plan to continue the investigation based on Hamiltonian (5) and analyse it in two aspects. Firstly, we calculate its partition function, and secondly we map it onto another Hamiltonian at operator level.

3. Calculation of the partition function

Expanding the two independent boson fields $\tilde{\phi}_0(x)$ and $\phi(x)$ as follows:

$$\tilde{\phi}_0(x) = \sum_k \frac{1}{\sqrt{L}}\frac{1}{\sqrt{2|k|}}(a_k\mathrm{e}^{\mathrm{i}kx} + a_k^\dagger\mathrm{e}^{-\mathrm{i}kx})$$
$$\phi(x) = \sum_k \frac{1}{\sqrt{L}}\frac{1}{\sqrt{2|k|}}(b_k\mathrm{e}^{\mathrm{i}kx} + b_k^\dagger\mathrm{e}^{-\mathrm{i}kx}) \quad (7)$$

in which a_k (b_k) and $a_k^\dagger$ ($b_k^\dagger$) satisfy the standard bosonic commutation relation, we obtain

$$\tilde{H}_b = (2\pi\rho)^{-1}\sum_k |k| a_k^\dagger a_k + \sum_k u_p|k| b_k^\dagger b_k$$
$$+\frac{j}{\sqrt{2\pi a}}\left\{\exp\left(\mathrm{i}\sqrt{4\pi}(1+\gamma)^{1/2}\sum_{k<0}\frac{1}{\sqrt{L}}\frac{1}{\sqrt{2|k|}}(a_k + a_k^\dagger)\right)\right.$$
$$\left.\times\exp\left(\mathrm{i}\frac{\lambda}{u_p}\sum_k\frac{1}{k\sqrt{L}}\frac{\sqrt{|k|}}{2}(b_k + b_k^\dagger)\right)d + \mathrm{h.c.}\right\}. \quad (8)$$

The above Hamiltonian can be divided as: $\tilde{H}_b = \tilde{H}_0 + \tilde{H}_1$. $\tilde{H}_0 = \tilde{H}_{01} + \tilde{H}_{02}$ contains two independent free fields, while $\tilde{H}_1$ describes *effective* hybridization between the local electron and conduction electrons under the influence of electron–electron and electron–phonon interactions. In the interaction picture (with β the inverse temperature), the partition function

$$Z = \mathrm{Tr}\left(\mathrm{e}^{-\beta\tilde{H}_0}T\mathrm{e}^{-\mathrm{i}\int_0^{-\mathrm{i}\beta}\tilde{H}_1(t)\mathrm{d}t}\right)$$

where $\tilde{H}_1(t) = \mathrm{e}^{\mathrm{i}\tilde{H}_0 t}\tilde{H}_1\mathrm{e}^{-\mathrm{i}\tilde{H}_0 t}$ and T is the time ordering operator. The partition function can be expanded as $Z = \sum_n Z_n$ with respect to $\tilde{H}_1$. When performing the trace over the

impurity's configurations, we find that Z_n vanishes for odd n, while for even n we obtain $Z_n = Z'_n + Z''_n$ with

$$Z'_n = \int_0^\beta \mathrm{d}\tau_n \int_0^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_0^{\tau_2} \mathrm{d}\tau_1 \,\mathrm{Tr}\left(\mathrm{e}^{-(\beta-\tau_n)\tilde{H}_{01}} A \mathrm{e}^{-(\tau_n-\tau_{n-1})\tilde{H}_{01}} A^\dagger \cdots A^\dagger \mathrm{e}^{-\tau_1 \tilde{H}_{01}}\right) \times \mathrm{Tr}\left(\mathrm{e}^{-(\beta-\tau_n)\tilde{H}_{02}} B \mathrm{e}^{-(\tau_n-\tau_{n-1})\tilde{H}_{02}} B^\dagger \cdots B^\dagger \mathrm{e}^{-\tau_1 \tilde{H}_{02}}\right) \tag{9}$$

with

$$A = \frac{j}{\sqrt{2\pi a}} \exp\left[\mathrm{i}\sqrt{4\pi}(1+\gamma)^{1/2} \sum_{k<0} \frac{1}{\sqrt{L}} \frac{1}{\sqrt{2|k|}} (a_k + a_k^\dagger)\right]$$

$$B = \exp\left[\mathrm{i}\frac{\lambda}{u_p} \sum_k \frac{1}{k\sqrt{L}} \frac{\sqrt{|k|}}{2} \left(b_k + b_k^\dagger\right)\right].$$

Here tracing has been performed in two independent subspaces because there exists no overlap between the two bosonic fields. The expression for Z''_n is obtained by interchanging $A \leftrightarrow A^\dagger$, $B \leftrightarrow B^\dagger$. It can be proved that $Z'_n = Z''_n$. If we define

$$A(t) = \mathrm{e}^{-\mathrm{i}t\tilde{H}_{01}} A \mathrm{e}^{\mathrm{i}t\tilde{H}_{01}} \qquad B(t) = \mathrm{e}^{-\mathrm{i}t\tilde{H}_{02}} B \mathrm{e}^{\mathrm{i}t\tilde{H}_{02}}$$

then

$$Z'_n = Z_0 \int_0^\beta \mathrm{d}\tau_n \int_0^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_0^{\tau_2} \mathrm{d}\tau_1 \langle A(\mathrm{i}\tau_n) A^\dagger(\mathrm{i}\tau_{n-1}) \cdots A^\dagger(\mathrm{i}\tau_1)\rangle \times \langle B(\mathrm{i}\tau_n) B^\dagger(\mathrm{i}\tau_{n-1}) \cdots B^\dagger(\mathrm{i}\tau_1)\rangle \tag{10}$$

where $Z_0 = \mathrm{Tr}(\mathrm{e}^{-\beta\tilde{H}_0})$. Now, we evaluate the thermodynamic expectation values using the strategies of Schotte [13] to obtain the partition function as

$$Z = 2Z_0 \left[1 + \sum_{n=1}^{\infty} \left(\frac{j}{\sqrt{2\pi a}}\right)^{2n} \int_0^\beta \mathrm{d}\tau_n \int_0^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_0^{\tau_2} \mathrm{d}\tau_1 \times \exp\left\{\sum_{\nu>\nu'}^{n} (-1)^{\nu+\nu'} \left(1 + \gamma + \frac{\lambda^2}{2\pi u_p^2}\right) \ln\left(\frac{\tau_\nu - \tau_{\nu'}}{\beta \bar{x}_0}\right)\right\}\right] \tag{11}$$

where $\bar{x}_0$ satisfy

$$\bar{x}_0^{\,1+\gamma+\frac{\lambda^2}{2\pi u_p^2}} = (v_F \beta k_0)^{-(1+\gamma)} (u_p \beta k_0')^{-\frac{\lambda^2}{2\pi u_p^2}}$$

and k_0 and k_0' are two fields' respective momentum cut-offs. The form of the partition function is the same as that of the following Kondo Hamiltonian

$$H_k = \sum_{\sigma,k} \hbar v_F k a_{k,\sigma}^\dagger a_{k,\sigma} + J_\parallel S^z s^z(0) + \tfrac{1}{2} J_\perp \left[S^+ s^-(0) + s^+(0) S^-\right] \tag{12}$$

with

$$J_\parallel = 2\rho^{-1} \left[1 - \frac{1}{\sqrt{2}} \left(1 + \gamma + \frac{\lambda^2}{2\pi u_p^2}\right)^{1/2}\right] \qquad J_\perp = 2(2\pi a)^{1/2} j. \tag{13}$$

In addition, if we set $\lambda = 0$, it will recover the partition function of the MCRL model, which is the same as [6]. For the present model, the quantity $\gamma + \frac{\lambda^2}{2\pi u_p^2}$ takes the place of γ in the MCRL model. We will discuss later that this will alter the phase transition condition.

4. Hamiltonian mapping at operator level

In this section we intend to further demonstrate the similarity between the current model and the MCRL model at operator level. First we transform the bosonic field $\phi(x)$ and its momentum field $\Pi(x)$ in (5) using a new bosonic field as follows

$$\begin{aligned}\phi(x) &= -\frac{1}{\sqrt{2}}\left[\phi'(x) - \int_{-\infty}^{-x}\Pi'(x')\mathrm{d}x'\right]\\ \Pi(x) &= -\frac{1}{\sqrt{2}}\left[\Pi'(x) + \frac{\partial}{\partial x}(\phi'(-x))\right]\end{aligned} \tag{14}$$

where $\phi'(x)$ and $\Pi'(x)$ satisfy $\left[\phi'(x), \Pi'(y)\right] = \mathrm{i}\delta(x-y)$. Now the two bosonic fields, which are contained in the exponent of the non-diagonal term in (5), take on the same form. Next we expect to modulate the velocities of the two free fields in order to make them identical, so we consider the following transformation

$$\begin{aligned}\phi'(x) &= \tilde{\phi}(\eta x)\\ \Pi'(x) &= \eta\tilde{\Pi}(\eta x)\end{aligned} \tag{15}$$

where η is a non-zero constant. It is easy to prove that $\left[\tilde{\phi}(x), \tilde{\Pi}(y)\right] = \mathrm{i}\delta(x-y)$. Under this transformation the quadratic term changes as

$$\int\left[\Pi'^2(x) + (\bigtriangledown\phi'(x))^2\right]\mathrm{d}x = \eta\int\left[\tilde{\Pi}^2(x) + (\bigtriangledown\tilde{\phi}(x))^2\right]\mathrm{d}x. \tag{16}$$

Adopting $\eta = (2\pi\rho u_p)^{-1}$, this scale stretching (for phonons only) will match the sound velocity to the Fermi velocity of the conduction electrons.

After (14) and (15), Hamiltonian (5) becomes

$$\begin{aligned}\tilde{H}_b = (4\pi\rho)^{-1}\Bigg\{&\int\left[\tilde{\Pi}_0^2(x) + (\bigtriangledown\tilde{\phi}_0(x))^2\right]\mathrm{d}x + \int\left[\tilde{\Pi}^2(x) + (\bigtriangledown\tilde{\phi}(x))^2\right]\mathrm{d}x\Bigg\}\\ &+\frac{j}{\sqrt{2\pi a}}\Bigg\{\exp\Bigg(\mathrm{i}\Bigg\{\sqrt{4\pi}(1+\gamma)^{1/2}\left[\tilde{\phi}_0(0) + \int_{-\infty}^{0}\tilde{\Pi}_0(x')\mathrm{d}x'\right]\\ &+\frac{\lambda}{\sqrt{2}u_p}\left[\tilde{\phi}(0) + \int_{-\infty}^{0}\tilde{\Pi}(x')\mathrm{d}x'\right]\Bigg\}\Bigg)d + \mathrm{h.c.}\Bigg\}.\end{aligned} \tag{17}$$

Finally, we combine fields $\tilde{\phi}_0(x)$ and $\tilde{\phi}(x)$ in the following form

$$\begin{aligned}\bar{\phi}(x) &= \frac{1}{\sqrt{c_1^2+c_2^2}}\left[c_1\tilde{\phi}_0(x) + c_2\tilde{\phi}(x)\right] \qquad \bar{\Pi}(x) = \frac{1}{\sqrt{c_1^2+c_2^2}}\left[c_1\tilde{\Pi}_0(x) + c_2\tilde{\Pi}(x)\right]\\ \bar{\bar{\phi}}(x) &= \frac{1}{\sqrt{c_1^2+c_2^2}}\left[-c_2\tilde{\phi}_0(x) + c_1\tilde{\phi}(x)\right] \qquad \bar{\bar{\Pi}}(x) = \frac{1}{\sqrt{c_1^2+c_2^2}}\left[-c_2\tilde{\Pi}_0(x) + c_1\tilde{\Pi}(x)\right]\end{aligned} \tag{18}$$

where $c_1 = \sqrt{\pi(1+\gamma)}$ and $c_2 = \lambda/\sqrt{2}u_p$. So we obtain

$$\begin{aligned}\tilde{H}_b = (4\pi\rho)^{-1}&\int\left[\bar{\Pi}^2(x) + (\bigtriangledown\bar{\phi}(x))^2\right]\mathrm{d}x\\ &+\frac{j}{\sqrt{2\pi a}}\left\{\exp\left(2\mathrm{i}\sqrt{c_1^2+c_2^2}\ \bar{\Phi}_L(0)\right)d + d^\dagger\exp\left(-2i\sqrt{c_1^2+c_2^2}\ \bar{\Phi}_L(0)\right)\right\}\end{aligned} \tag{19}$$

where

$$\bar{\Phi}_L(x) = \frac{1}{2}\left[\bar{\phi}(x) + \int_{-\infty}^{x} \bar{\Pi}(x')\mathrm{d}x'\right].$$

In (19) we have omitted the free field

$$\int\left[\overline{\overline{\Pi}}^2(x) + \left(\nabla\,\overline{\overline{\phi}}(x)\right)^2\right]\mathrm{d}x.$$

Up to now the form of (19) has been the same as that of (6). i.e., we relate the current Hamiltonian to the usual MCRL Hamiltonian. One can easily find their differences: the quantity $\sqrt{4\pi}(1+\gamma)^{1/2}$ in (6) is now changed into $2\sqrt{c_1^2+c_2^2}$ in (19), i.e.

$$\sqrt{4\pi}(1+\gamma)^{1/2} \longrightarrow 2\sqrt{c_1^2+c_2^2}.$$

It is equivalent to $\gamma \longrightarrow \gamma + \frac{\lambda^2}{2\pi u_p^2}$. This conclusion is exactly the same as that in section 3. Applying a further canonical transformation

$$U_2 = \exp\left[-\mathrm{i}\left(\sqrt{8\pi} - 2\sqrt{c_1^2+c_2^2}\right)\bar{\Phi}_L(0)(d^\dagger d - \tfrac{1}{2})\right]$$

Hamiltonian (19) can be mapped onto the anisotropic Kondo model defined in (12) and (13).

Here we further explore the effects of the interaction between the phonons and the local electrons on the dynamics of the conduction electrons. In fact, the phonon interaction in Hamiltonian (1) takes a similar form to the screening conduction electrons, i.e., both the phonon and the electron–hole pairs are scattered by the local electron at the origin. Hence the phonons also take part in the combination of the electron–hole pairs, and contribute to γ with their own phase shift.

5. Discussions and conclusions

In the above treatment, we relate the model including local electron–phonon interactions to the usual MCRL model, so we can conveniently discuss the effects of phonons on the FL–non-FL transition. From (12) and (13), one can see that, for $\gamma + \frac{\lambda^2}{2\pi u_p^2} < 1$, the model corresponds to an antiferromagnetic Kondo problem, and the system then takes on FL behaviour; for $\gamma + \frac{\lambda^2}{2\pi u_p^2} > 1$, the model corresponds to a ferromagnetic Kondo problem, and the system then takes on non-FL behaviour. Now the transition point is at $\gamma_c = 1 - \lambda^2/2\pi u_p^2$, which is different from $\gamma = 1$ in the case of the usual MCRL model. Obviously, in the current model, the transition point moves to the left, i.e., the non-FL region expands and the FL region shrinks. This result may be understood qualitatively from the following picture. By including the local electron–phonon interactions, the local electron will fluctuate less easily compared with the case where there is only local electron–electron interaction, because its fluctuation is accompanied by local lattice distortion, just like the enhancement of effective mass in the polaron problem [14]. In fact, the local electron–phonon interaction is equivalent to an additional screening channel (see (3)), therefore the phase transition point moves towards the FL region. We emphasize here, however, that the local electron–phonon interaction plays the role of an *anomalous* screening channel because there is no upper limit π on its phase shift, so we may expect its non-trivial influence on the properties of the MCRL model. This situation is similar to the problem of a two-state system coupled to a phonon bath or a fermion bath. Although a fermion bath can be mapped onto a phonon

bath via bosonization techniques, it is known that there exists a broken-symmetry transition for a phonon bath, while there is no such transition for a fermion bath [16]. From the definition of γ we can re-evaluate the necessary channel number. In the case of sufficiently strong screening, the phase shift δ_l equals π, then $\gamma = N - 2$, where N is the total channel number. The necessary condition forcing the system to take on non-FL behaviour is: $N - 2 + \frac{\lambda^2}{2\pi u_p^2} > 1$. We find the minimum channel number decreases, and it need not be three as in the case of non-phonon interactions. As an example, for strong local electron–phonon interaction, $\lambda > \lambda_c = (4\pi u_p^2)^{1/2}$, we need only *one* channel in order to realize the FL–non-FL transition. Therefore, *it is the local electron–phonon interaction, rather than the local electron–electron interaction, that determines the behaviour of the system*. Moreover, we want to point out that the phonon-induced FL–non-FL transition takes place only at very low temperature. In fact, in the current problem there exists two scales: the Fermi energy ϵ_F and the Debye frequency ω_D, so there exists a characteristic temperature $T^*(\omega_D)$. When $T > T^*(\omega_D)$ the phonons make no contribution to the renormalization of the hybridization parameter j, therefore, for sufficiently small γ the system will take on FL behaviour; while when $T < T^*(\omega_D)$ the phonons affect renormalization of j and the system may take on non-FL behaviour.

We can investigate the properties of the current model with help of the known results of the SC Kondo model and its related models, such as the dissipative two-state system [17]. The first property under consideration is the contribution of the local electron to the specific heat ΔC at low temperature. For $\gamma + \frac{\lambda^2}{2\pi u_p^2} < 1$, we have the linear relation $\Delta C \sim T$, while for $2 > \gamma + \frac{\lambda^2}{2\pi u_p^2} > 1$, the specific heat varies non-linearly with the temperature

$$\Delta C \sim T^{\gamma + \frac{\lambda^2}{2\pi u_p^2} - 1}.$$

The second property is the dynamics about $P(t) = \langle n_d(t) \rangle$ of the local impurity at low temperature, assuming that at time $t = 0$ the local orbital is occupied by an electron $n_d(0) = 1$. For $0 < \gamma + \frac{\lambda^2}{2\pi u_p^2} < 1$, $P(t)$ is described by damped relaxation, while for $\gamma + \frac{\lambda^2}{2\pi u_p^2} > 1$, $P(t) = 1$. The transition from FL to non-FL behaviour at $\gamma_c = 1 - \frac{\lambda^2}{2\pi u_p^2}$ can be seen clearly from the above results.

Before drawing conclusions, we want to point out the power-law dependence of the specific heat on temperature $\Delta C \sim T^\delta$ with $\delta \simeq 2/3$ has been observed in the heavy-fermion system UPd_xCu_{5-x} with $x = 1$ and $x = 1.5$ [2]. This unusual non-FL behaviour deviates from the prediction of the two-channel Kondo model and it may be qualitatively described by the present model. Recently, a three-channel resonant-level model with spin has been proposed for non-FL behaviour in heavy-fermion systems [18] and further work is needed to study the effects of phonons.

In conclusion, we have discussed the phonon effects on the MCRL model and found that the model still undergoes the FL–non-FL transition when there is local electron–phonon interaction, but the non-FL region expands and the FL region shrinks in comparison with the usual MCRL model. In particular, for strong local electron–phonon interaction we need only one channel in order to realize the FL–non-FL transition, and it is the phonon effects that control the behaviour of the system.

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Appendix

In this section we discuss how to describe the local electron–phonon interaction. Although in real physical problems the phonons are three dimensional, here we want to demonstrate that the three-dimensional phonons can be substituted for one-dimensional phonons allowing for their equivalence as far as their contributions to the partition function are concerned. The real local electron–phonon interaction can be described as (the local electron is located at the zero point):

$$\tilde{H}_{e-p} = (d^\dagger d - \tfrac{1}{2}) \sum_j \boldsymbol{Q}_j \cdot \nabla V(\boldsymbol{R}_j^{(0)}). \tag{A1}$$

Here V denotes the potential function generated by an individual ion and $\boldsymbol{R}_j^{(0)}$ is the equilibrium position of the ion. $\boldsymbol{Q}_j$ describes the displacement of the jth ion, which can be expanded as [15]

$$\boldsymbol{Q}_j = \sum_{\boldsymbol{k},\sigma} \frac{1}{\sqrt{2NM\omega_{\boldsymbol{k},\sigma}}} \xi_{\boldsymbol{k},\sigma} (b_{\boldsymbol{k},\sigma} + b^\dagger_{-\boldsymbol{k},\sigma}) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R}_j^{(0)}}. \tag{A2}$$

Here M is the ion mass, $\xi_{\boldsymbol{k},\sigma}$ is the polarization vector which specifies the vibrational direction of the ion for each wave vector and σ. The index σ runs over those $3n$ values of the normal mode, where n is the number of atoms per unit cell. Each mode has its own eigenfrequency $\omega_{\boldsymbol{k},\sigma}$. If we assume the local electron only interacts with the nearest-neighbouring ions and only retain longitudinal acoustic phonons (σ is dropped), in the long-wavelength limit we obtain

$$\tilde{H}_{e-p} = (d^\dagger d - \tfrac{1}{2}) \sum_{\boldsymbol{k}} g_{\boldsymbol{k}} (b_{\boldsymbol{k}} + b^\dagger_{-\boldsymbol{k}}) \tag{A3}$$

where $g_{\boldsymbol{k}} = C/\sqrt{2NM|\boldsymbol{k}|}$ and $C = \sum_j \xi_{\boldsymbol{k}} \cdot \nabla V(\boldsymbol{R}_j^{(0)})$ is a constant. When we calculate the partition function, as we have done in section 3, the phonons' contribution is only decided by the spectrum density $J(\omega) = \sum_{\boldsymbol{k}} g_{\boldsymbol{k}}^2 \delta(\omega - \omega_{\boldsymbol{k}})$ [17], which depends on the dimension of k space and the coupling coefficient $g_{\boldsymbol{k}}$. For current three-dimensional phonons with coupling coefficient $g_{\boldsymbol{k}} \sim |\boldsymbol{k}|^{-1/2}$, obviously we can give the same spectrum density if we consider a kind of one-dimensional phonon with a different coupling coefficient $g_k \sim |k|^{1/2}$, i.e., we can write local electron–phonon interaction in the form

$$H_{e-p} = (d^\dagger d - \tfrac{1}{2}) \sum_k g_k (b_k + b^\dagger_{-k}). \tag{A4}$$

Note here the relation of the coupling coefficient g_k about the mode of k has changed. In summary, if the dimension of the phonons is transformed from three to one and the relation of the coupling coefficient g_k about the mode of k is transformed accordingly, the spectrum density $J(\omega)$ will be reproduced, and subsequently so will the partition function. So far we obtain the form in equation (1).

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